

Math 217, Probability and Statistics

First Test

1 Oct 2007

Instructor: _____ Your name: _____

You may refer to one sheet of notes on this test, and you may use a calculator. You may leave your answers as expressions such as $\binom{8}{4} \frac{e^{1/3}}{\sqrt{2\pi}}$ if you like.

Problem 1. [12] Give an example of a probability density function f for a nonuniform continuous random variable. (You get to choose which one; just make sure it's not a uniform one.)

Problem 2. [12] A fair die is tossed 6 times. What is the probability that exactly three 5's appear?

Problem 3. [16; 4 points each part] For each of the following functions $F(x)$, could F be a cumulative distribution function for a continuous random variable? You don't have explain your answer; just write "yes" or "no".

_____ **a.** $F(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ x/2 & \text{if } 0 < x < 2 \\ 1 & \text{if } 2 \leq x \end{cases}$

_____ **b.** $F(x) = \begin{cases} 0 & \text{if } x \leq 2 \\ 1/8 & \text{if } 2 < x < 10 \\ 0 & \text{if } 10 \leq x \end{cases}$

_____ **c.** $F(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 - e^{-x} & \text{if } 0 < x \end{cases}$

_____ **d.** $F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{2} & \text{if } 0 \leq x < 1 \\ 1 & \text{if } 1 \leq x \end{cases}$

Problem 4. [12; 6 points each part] A dart is thrown at a circular dartboard of radius 10 inches. Assume that the location that the dart lands is a uniform continuous distribution, that is, the probability that it lands in a region of the dartboard is proportional to the area of that region.

(a). What is the probability that the dart falls within 5 inches of the center of the target?

(b). What is the probability that the dart falls within 3 inches of the edge of the target?

Problem 5. [12] Two fair dice are tossed, one red and one green. What is the probability that they show the same number? Explain your answer in terms of outcomes in a sample space.

Problem 6. [12] Four cards are dealt from a standard deck of cards (4 suits, 13 cards in each suit). What is the probability that one of the four cards is in each suit (that is, no two of the four cards are in the same suit). Explain your reasoning.

Problem 7. [12] Suppose a number X is chosen randomly and uniformly from the interval $[-10, 10]$. Determine $P(X^2 > 4)$.

Problem 8. [12] Recall that a set Ω is said to be *partitioned* into subsets A_1, A_2, \dots, A_n if each element of Ω belongs to exactly one of the subsets A_i . Let $m : \Omega \rightarrow [0, 1]$ be a discrete probability distribution on Ω . Prove that if Ω is partitioned into the events A_1, A_2, \dots, A_n then

$$P(A_1) + P(A_2) + \dots + P(A_n) = 1.$$

Base your proof on outcomes in the sample space Ω .

#1.[12]	
#2.[12]	
#3.[16]	
#4.[12]	
#5.[12]	
#6.[12]	
#7.[12]	
#8.[12]	
Total	