1. Choose a number \( B \) at random from the interval \([0, 1]\) with uniform density. Find the probability that
   a. \( \frac{1}{3} < B < \frac{2}{3} \),
   b. \(|B - \frac{1}{2}| \leq \frac{1}{4} \),
   c. \( B < \frac{1}{4} \) or \( 1 - B < \frac{1}{4} \),
   d. \( 3B^2 < B \).

2. Let \( x \) and \( y \) be chosen uniformly and independently at random from the interval \([0, 1]\). Which pairs of the following events are independent?
   a. \( x > \frac{1}{3} \).
   b. \( y > \frac{2}{3} \).
   c. \( x > y \).
   d. \( x + y < 1 \).

3. Let \( X \) be a random variable with cumulative distribution function \( F \). The median of \( X \) is the value \( m \) for which \( F(m) = \frac{1}{2} \). Then \( X < m \) with probability \( \frac{1}{2} \) and \( X > m \) with probability \( \frac{1}{2} \). Find \( m \) if \( X \) is
   a. uniformly distributed over the interval \([a, b]\),
   b. exponentially distributed with parameter \( \lambda \).

4. Let \( X \) be a random variable normally distributed with parameters \( \mu = 70, \sigma = 10 \). Estimate
   a. \( P(X > 50) \).
   b. \( P(X < 60) \).
   c. \( P(X > 90) \).
   d. \( P(60 < X < 80) \).

5. A final examination at Podunk University is constructed so that the test scores are approximately normally distributed, with parameters \( \mu \) and \( \sigma \). The instructor assigns letter grades to the test scores as shown below (this is the process of “grading on the curve”).

<table>
<thead>
<tr>
<th>Test Score</th>
<th>Letter grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu + \sigma &lt; x )</td>
<td>A</td>
</tr>
<tr>
<td>( \mu &lt; x &lt; \mu + \sigma )</td>
<td>B</td>
</tr>
<tr>
<td>( \mu - \sigma &lt; x &lt; \mu )</td>
<td>C</td>
</tr>
<tr>
<td>( \mu - 2\sigma &lt; x &lt; \mu - \sigma )</td>
<td>D</td>
</tr>
<tr>
<td>( x &lt; \mu - 2\sigma )</td>
<td>F</td>
</tr>
</tbody>
</table>

What fraction of the class gets A, B, C, D, F?

6. Let \( X_1 \) and \( X_2 \) be independent random variables with common distribution
   \[
   p_X = \begin{pmatrix} 0 & 1 & 2 \\ 1/8 & 3/8 & 1/2 \end{pmatrix}
   \]
   Find the distribution of the sum \( X_1 + X_2 \).

7. Let \( X \) and \( Y \) be independent random variables defined on the space \( \Omega \), with density functions \( f_X \) and \( f_Y \), respectively. Suppose that \( Z = X + Y \). Find the density \( f_Z \) of \( Z \) if
   \[
   f_X(x) = f_Y(x) = \begin{cases} 
   1/2, & \text{if } -1 \leq x \leq +1, \\
   0, & \text{otherwise.}
   \end{cases}
   \]

8. Suppose again that \( Z = X + Y \). Find \( f_Z \) if
   \[
   f_X(x) = f_Y(x) = \begin{cases} 
   x/2, & \text{if } 0 < x < 2, \\
   0, & \text{otherwise.}
   \end{cases}
   \]