CLARK UNIVERSITY 1887

Eighth assignment
Math 217 Probability and Statistics

Prof. D. Joyce, Fall 2014

1. (Exercise 6.47) Consider a sample of size 5 from a uniform distribution over $[0,1]$. Compute the probability that the median lies in the interval $[\frac{1}{4}, \frac{3}{4}]$.

2. (Exercise 6.48) If X_1, X_2, X_3, X_4, X_5 are independent and identically distributed exponential random variables with the parameter λ, compute

a. $P(\min(X_1, \ldots, X_5) \leq a)$ where a is a positive constant.

b. $P(\max(X_1, \ldots, X_5) \leq a)$.

3. (Exercise 6.52) Let X and Y denote the coordinates of a point chosen uniformly at random in the unit circle. Then the joint density function $f(x,y)$ is constantly $1/\pi$ when $x^2 + y^2 \leq 1$, and 0 otherwise.

a. Show that the Jacobian for the change to polar coordinates is

$$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = r.$$

You'll probably recognize that Jacobian from change of coordinates formula. $dx\,dy = r\,dr\,d\theta$.

b. Find the joint density function for the polar coordinates $R = \sqrt{X^2 + Y^2}$, $\Theta = \arctan \frac{Y}{X}$.

4. (Exercise 6.56a) If X anad Y are independent and identically distributed uniform random variables on $[0,1]$, compute the joint density of $U = X + Y$, $V = X/Y$.

5. (Exercise 7.4) If X and Y have the joint density function

$$f(x,y) = \begin{cases} 1/y & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

find

a. $E(XY)$

b. $E(X)$

c. $E(Y)$

6. (Exercise 7.30) Let X and Y be independent and identically distributed random variables with mean μ and variance σ^2. Find $E((X-Y)^2)$.

7. (Exercise 7.38) Let random variables X and Y have joint density

$$f(x,y) = \begin{cases} 2e^{-x}/x & \text{if } 0 \leq y \leq x \\ 0 & \text{otherwise} \end{cases}$$

Compute $\text{Cov}(X,Y)$.

8. (Exercise 7.45a) Let X_1, X_2, X_3 be pairwise uncorrelated random variables, that is, any pair of them have correlation 0, and let each of them have mean 0 and variance 1. Compute the correlations of $X_1 + X_2$ and $X_2 + X_3$.

Math 217 Home Page at `http://math.clarku.edu/~djoyce/ma217/`