Conditional distributions
Math 217 Probability and Statistics
Prof. D. Joyce, Fall 2014

Suppose you have joint distributions $X$ and $Y$ and denote their joint cumulative distribution function by $F(x,y)$ and their joint probability mass or density function by $f(x,y)$. Their marginal cumulative distribution functions are $F_X(x)$ and $F_Y(y)$ and their probability functions are $f_X(x)$ and $f_Y(y)$ as we saw before.

There are also conditional distribution and probability functions.

The conditional cumulative distribution function for $X$ given that $Y$ has the value $y$ is denoted in various ways. Our text denotes it $F_{X|Y}(x|y)$. Likewise, the corresponding conditional probability mass or density function is denoted $f_{X|Y}(x|y)$. There are also conditional functions for $Y$ given that $X$ has a value $x$.

As you would expect, if $X$ and $Y$ are independent, then the conditional probability functions are the same as the marginal functions:

$$f_{X|Y}(x|y) = f_X(x).$$

**The definition in the continuous case.** For the continuous case the conditional cumulative distribution function $F_{X|Y}(x|y)$ is

$$F_{X|Y}(x|y) = P(X \leq x|Y=y) = \frac{1}{f_Y(y)} \int_{-\infty}^{x} f(t,y) \, dx$$

Its derivative with respect to $x$ gives the conditional probability density function $f_{X|Y}(x|y)$ for $X$ given $Y$ has the value $y$ is

$$f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)}.$$

**The definition in the discrete case.** Here, $X$ and $Y$ are joint discrete random variables. The conditional cumulative distribution function $F_{X|Y}(x|y)$ is

$$F_{X|Y}(x|y) = P(X \leq x|Y=y) = \frac{1}{f_Y(y)} \sum_{t \leq x} f(t,y)$$

The conditional probability mass function $f_{X|Y}(x|y)$ for $X$ given $Y$ has the value $y$ is

$$f_{X|Y}(x|y) = P(X=x|Y=y) = \frac{f(x,y)}{f_Y(y)}.$$