Math 218 Mathematical Statistics

First Test Answers

20 Feb 2009

Problem 1. [16; 4 points each part] Briefly explain these terms of experimental design. One sentence should do for each.

a. Treatment factors.

Treatment factors are those factors (predictor variables) that can be controlled (set) by the experimenter.

b. Noise factors, also called nuisance factors.

These are the factors that might affect the response variable but are not controlled by the experimenter.

c. Level of a treatment factor.

They're the different levels (values) that the experimenter sets the treatment factors.

d. Treatment.

A treatment is a particular combination of the levels of the different treatment factors; if there's only one factor, then it's just the treatment level.

Problem 2. [20; 10 points each part] A truck can carry a maximum load of 4000 lb. A manufacturer wants to ship an order of 50 boxes. The weights of the boxes are normally distributed with mean $\mu = 78$ lb. and standard deviation $\sigma = 12$ lb.

a. What is the probability that all 50 boxes can be sent in one shipment?

Here are two methods of answering the question, the first using the sample sum S_n, the second using the sample mean $\overline{X}$. Either one, when normalized, can be used to find the probability.

Method 1. Using the sample sum S_n, we need to determine $P(S_n \leq 4000)$. Since the weight of each box is normally distributed with mean $\mu = 78$ and standard deviation $\sigma = 12$, therefore the sample sum is also normally distributed but with mean $\mu_{S_n} = 50\mu = 3900$ and standard deviation $\sigma_{S_n} = \sqrt{n}\,\sigma = \sqrt{50} \cdot 12 = 84.85$. Since 4000 is 100 greater than the mean 3900, and 100 is $100/84.85 = 1.18$ standard deviations, we need to find $P(Z \leq 1.18)$ where Z is a standard normal distribution, that is, $\Phi(1.18)$.

Method 2. Using the sample mean $\overline{X}$, we need to determine $P(\overline{X} \leq 4000/50)$, that is, $P(\overline{X} \leq 80)$. Since the weight of each box is normally distributed with mean $\mu = 78$ and standard deviation $\sigma = 12$, therefore the sample mean is also normally distributed with the same mean $\mu_{S_n} = \mu = 78$ but with standard deviation $\sigma_{S_n} = \sigma/\sqrt{n} = \cdot 12/\sqrt{50} = 1.697$. Since 80 is 2 greater then the mean 78, and 2 is $2/1.697 = 1.18$ standard deviations, we again need $\Phi(1.18)$.

b. If the weights are not normally distributed, will the answer be still approximately correct? Why or why not?

The answer will still be approximately correct because with n as large as 50, both the sample sum and the sample mean will be very close to normal distributions with the right means and standard deviations. (Central Limit Theorem.)

Problem 3. [18; 3 points each part] Data variables come in various types. For each of the following, decide whether it is nominal, ordinal, continuous, or discrete. (If it's unclear, explain in a short sentence why.)

A handbook lists the following information on accredited U.S. colleges and university:

a. enrollment
discrete

b. required entrance tests (ACT, SAT, or none)
nominal

c. annual tuition
discrete

d. fields of study
nominal

e. selectivity (high, moderate, low)
ordinal

f. percent of applicants accepted
continuous (if fractional percentages are reported)

Problem 4. [15; 3 points each part] True or false. Write the whole word "true" or the whole word "false." (If it's not clear whether it's true or false, you may write a short explanation.)

a. A statistical study that compares two groups or methods is called a historical study.

False. That's a comparative study.

b. Data variables come in two different types: (1) categorical variables, also called qualitative variables, which always have the same value for all experiments, and (2) numerical variables, also called quantitative variables, which have different values depending on the outcome of the experiment.

False. Categorical variables have values that change with the experiment. They just aren't numerical.

c. Three different kinds of experimental error are (1) systematic error, which is caused by the differences between experimental units, (2) random error, which is caused by the inherent variability in the responses of similar experimental units given the same treatment, and (3) measurement error, which is caused by imprecise measuring instruments.

True.

d. The difference between a sample and a census is that all the individuals are surveyed in a census while only some of the individuals are surveyed in a sample.

True.

e. A χ^2- distribution with 10 degrees of freedom is the distribution that the sum of 10 independent standard normal random variables has.

False. It's sum of 10 *squares* of independent standard normal random variables

Problem 5. [15; 5 points each part] Consider the χ^2 distribution with 8 degrees of freedom.

a. Using table A.5, find constants $a, b, c, d,$ and e so that $P(\chi^2_8 > a) = 0.05$, $P(\chi^2_8 > b) = 0.99$, $P(\chi^2_8 < c) = 0.90$, and $P(d < \chi^2_8 < e) = 0.95$.

$a = 15.507$.

$b = 1.646$.

We need $P(\chi^2_8 > c) = 0.10$. So $c = 13.372$.

For d and e is reasonable to split the two ends evenly. We need $(\chi^2_8 > d) = 0.975$. So $d = 2.180$.

We need $\chi^2_8 < e) = 0.025$. So $e = 17.534$.

b. Express the constants from part **a** in terms of the notation $\chi^2_{\nu,\alpha}$.

$a = \chi^2_{8,.05}$

$b = \chi^2_{8,.99}$

$c = \chi^2_{8,.1}$

$d = \chi^2_{8,.975}$

$e = \chi^2_{8,.025}$

c. Sketch each of the probabilities from part **b** as an area under a χ^2 probability density curve.

Problem 6. [16] Short essay. Explain the importance of a control group in a comparative study. First say what a control group is. (You may want to give an example to clarify the concept.) Then describe the advantages that an experiment that has both a control group and one or more treatment groups has over an experiment that has only treatment groups.

The goal of most comparative studies is to evaluate how a condition affects a response variable. The normal condition is the control and is used to see the effects of a treatment. For instance, to see if a certain insecticide works to kill insects on plants, two groups of plants with insects are monitored under the all the same conditions except that the insecticide is used only on one group; the control group doesn't get the insecticide. Thus, various other factors such as sunlight, temperature, water, etc. are the same, so the effects of these nuisance factors (also called confounding variables) are minimized.

You could also mention various other aspects of control groups such as placebos, single blind studies, double blind studies, etc., but this is a short essay, so they aren't required.