Math 218 Mathematical Statistics

Second Test

25 March 2008

Your name: ____________________________

You may refer to one sheet of notes on this test, and you may use a calculator. You may leave your answers as expressions such as $\binom{8}{4}\frac{e^{1/3}}{\sqrt{2\pi}}$ if you like. Points for each problem are in square brackets.

Problem 1. [20; 10 points each part] Recall that the margin of error E of a confidence interval $[\overline{x} - z_{\alpha/2}\,\frac{\sigma}{\sqrt{n}}, \overline{x} + z_{\alpha/2}\,\frac{\sigma}{\sqrt{n}}]$ is half the width of the interval, that is, $E = z_{\alpha/2}\,\frac{\sigma}{\sqrt{n}}$.

a. If the level of significance α is fixed, in order to reduce the margin of error E by a factor of 10, does n have to increase or decrease, and by what factor?

b. If the level of significance α is decreased from 0.10 to 0.05 while the sample size n is left fixed, is the margin of error increased or decreased, and by what factor?

Problem 2. [30; 10 points each part] Exercise 6.28, page 234. In 1993 a typical American family spent 22% of its after-tax income on cloting, entertainment, and other activities, while the remaining 78% was spent on essentials like housing, food, transportation, health care, and insurance/pensions. It is suggested that because of the increase in the cost of essentials since 1993, the mean percentage income spent on the first three items has decreased. To test this proposition a random sample of 50 households is taken and the percentage spent on these three items is determined. Assume that the percentage varies across families according to a normal distribution with unknown mean μ and known $\sigma = 5\%$.

a. Set up hypothesis on μ. Clearly, H_0 should be $\mu = 22\%$. State what you think H_1 should be (upper one-sided, lower one-sided, or two-sided) and in one sentence explain why.

b. If the average percentage for the random sample of 50 households is 20.5%, would you conclude that this is a significant decrease compared to the 1993 level at the 99% confidence level, i.e., using the level of significance $\alpha = 0.01$? (Show your work.)

c. Given that average percentage is 20.5%, what is the P-value, also called the observed level of significance?

Problem 3. [30; 10 points each part] Frequently we have used the statistic

$$Z = \frac{\overline{X} - \mu}{S/\sqrt{n}}$$

in the use of inferences about an unknown population mean μ. (Here, $\overline{X}$ is the sample mean, S is the sample standard deviation, and n is the size of the sample.)

a. What are the two approximations that justify the claim that Z is approximately standard normal?

b. What theorem is used to justify one at least one of those claims?

c. What assumption or assumptions are necessary for the justification you gave in part b?

Problem 4. [20] Exercises 8.7 and 8.16. In a matched pairs design test, to determine whether glaucoma affects the corneal thickness, measurements were made in 8 people affected by glaucoma in one eye but not in the other. The corneal thicknesses in microns were as follows.

Person	1	2	3	4	5	6	7	8
Eye affected by glaucoma	488	478	480	426	440	410	458	460
Eye not affected by glaucoma	484	478	492	444	436	398	464	476
Difference	4	0	−12	−18	4	12	−6	−16

From this data the following statistics can be computed. The mean value for the eye affected by glaucoma is $\overline{x} = 455$ while the mean value for the eye unaffected by glaucoma is $\overline{y} = 459$, so the mean difference is $\overline{d} = \overline{x} - \overline{y} = -4$. The sample standard deviation on the difference works out to be $s_d = 10.74$.

Test $H_0 : \mu_1 = \mu_2$ against a two-sided alternative using $\alpha = .10$. What do you conclude? Do you reject H_0 or not? (Show your work.)

#1.[20]	
#2.[30]	
#3.[30]	
#4.[20]	
Total	