The Book Review Column[1]
by Frederic Green

Department of Mathematics and Computer Science
Clark University
Worcester, MA 01610
email: `fgreen@clarku.edu`

The first two reviews in this column draw from the "SpringerBriefs" series, dedicated to compact summaries of cutting-edge research in a variety of fields. The first is from SpringerBriefs in Physics, the second from SpringerBriefs in Applied Sciences and Technology. We close with a review about algorithms in the context of the modern world.

1. **Three Lectures on Complexity and Black Holes**, by Leonard Susskind. An introduction to exciting recent connections between quantum computational complexity and black holes. Review by Frederic Green.

2. **A Short Course in Computational Geometry and Topology**, by Herbert Edelsbrunner. An accessible introduction that uses computational methods to motivate and shed light on more general ideas in geometry and topology, and also many aspects of current research. Review by Abdulai Gassama and Frederic Green.

3. **The Age of Algorithms,** by Serge Abiteboul and Gilles Dowek. A non-technical introduction to algorithms, as well as with their implications (for good or ill) in contemporary society. Review by S. V. Nagaraj.

As always, please contact me to write a review; choose from among the books listed on the next pages, or, if you are interested in anything not on the list, just send me a note.

BOOKS THAT NEED REVIEWERS FOR THE SIGACT NEWS COLUMN

Algorithms

1. *Algorithms and Data Structures Foundations and Probabilistic Methods for Design and Analysis*, by Helmut Knebl
2. *Algorithms and Data Structures*, by Helmut Knebl
3. *Beyond the Worst-Case Analysis of Algorithms*, by Tim Roughgarden

Computability, Complexity, Logic

1. *Applied Logic for Computer Scientists: Computational Deduction and Formal Proofs*, by Mauricio Ayala-Rincón and Flávio L.C. de Moura.
2. *Descriptive Complexity, Canonisation, and Definable Graph Structure Theory*, by Martin Grohe.
3. *Semigroups in Complete Lattices*, by P. Eklund, J. Gutiérrez García, U. Höhle, and J. Kortelainen.

Miscellaneous Computer Science

1. *Elements of Causal Inference: Foundations and Learning Algorithms*, by Jonas Peters, Dominik Janzing, and Bernhard Schölkopf.
2. *Partially Observed Markov Decision Processes,* by Vikram Krishnamurthy
3. *Statistical Modeling and Machine Learning for Molecular Biology*, by Alan Moses
4. *Language, Cognition, and Computational Models,* Theirry Poibeau and Aline Villavicencio, eds.
5. *Computational Bayesian Statistics, An Introduction,* by M. Antónia Amaral Turkman, Carlos Daniel Paulino, and Peter Müller.
6. *Variational Bayesian Learning Theory,* by Shinichi Nakajima, Kazuho Watanabe, and Masashi Sugiyama.
7. *Knowledge Engineering: Building Cognitive Assistants for Evidence-based Reasoning*, by Gheorghe Tecuci, Dorin Marcu, Mihai Boicu, and David A. Schum.
8. *Quantum Computing: An Applied Approach*, by Jack D. Hidary

Discrete Mathematics and Computing

1. *Mathematics in Computing*, by Gerard O'Regan
2. *Understand Mathematics, Understand Computing – Discrete Mathematics That All Computing Students Should Know*, by Arnold L. Rosenberg and Denis Trystram

Cryptography and Security

1. *Computer Security and the Internet: Tools and Jewels*, by Paul C. van Oorschot

Combinatorics and Graph Theory

1. *The Zeroth Book of Graph Theory: An Annotated Translation of Les Réseaux (ou Graphes) – André Sainte-Laguë (1926)*, translated by Martin Charles Golumbic
2. *Finite Geometry and Combinatorial Applications*, by Simeon Ball
3. *Combinatorics, Words and Symbolic Dynamics,* Edited by Valérie Berthé and Michel Rigo

Programming etc.

1. *Formal Methods: An Appetizer*, by Flemming Nielson and Hanne Riis Nielson
2. *Sequential and Parallel Algorithms and Data Structures*, by P. Sanders, K. Mehlhorn, M. Dietzfelbinger, R. Dementiev

Miscellaneous Mathematics

1. *Introduction to Probability*, by David F. Anderson, Timo Seppäläinen, and Benedek Valkó.
2. *Algebra and Geometry with Python*, by Sergei Kurgalin and Sergei Borzunov.

Review of[2]
Three Lectures on Complexity and Black Holes
by Leonard Susskind
Springer, 2020
100 pages, Softcover, $59.99.

Review by
Frederic Green (fgreen@clarku.edu)
Departments of Mathematics and Computer Science
Clark University, Worcester, MA

The future prospects for anyone falling into a black hole are bleak. For one thing, there is no chance (according to our present state of knowledge) of ever getting out again. Worse, one is facing certain destruction when one meets the "singularity" (or its inconceivably dense physical manifestation, whatever that may be) inside. However, there is an "event horizon," the point of no return, separating the overly curious infalling astronaut from the doom he or she faces at the singularity. Suppose Alice the Astronaut wants to see what's behind the horizon (never mind the consequences). How much time would Alice have to look around and see what's happening, before reaching the end of her worldline? Conventional wisdom, until relatively recently, was that she would have some amount of time, perhaps hours. Passing the event horizon of a supermassive black hole would not seem like any kind of a milestone to the infalling individual; it is only an outside observer who would notice something out of the ordinary.

Given the fact that no experimenters will be returning from probing the interior of a black hole, understanding what happens beyond the event horizon, and indeed the nature of the event horizon itself, is far from an academic exercise. It turns out that this relates to deep questions such as the second law of thermodynamics and the quantization of Einsteinian gravity.

Of course, there is also the obvious question of why the previous two paragraphs are appearing on the pages of SIGACT News. The answer is simple (albeit conjectural), according to this fascinating little book by Leonard Susskind: The evolution of the interior of a black hole is deeply connected to computation. In fact, as I will describe below, in this book many notions of great interest to theoretical computer scientists seem to be key to understanding these phenomena: e.g., quantum circuits, quantum complexity, and expander graphs.

1 Complexity

This book is in three parts, corresponding to three lectures Susskind gave at the PiTP[3] Summer Program at IAS in 2018 (see [Su]; they are well worth watching!). Part I ("Hilbert Space is Huge", 9 chapters) addresses the purely complexity and group theoretic aspects of this research. Part II ("Black Holes and the Second Law of Complexity," 8 chapters) conjectures a relation between the growth of complexity in the course of a computation, and the growth of the region behind the event horizon. Part III ("The Thermodynamics of Complexity," 6 chapters) investigates more deeply an analog between the second law of thermodynamics and a similar law governing the evolution of complexity. The individual chapters in each part are quite short, taking up anywhere from 2 to 9 pages.

[3]Prospects in Theoretical Physics

What the title of Part I asserts, that Hilbert space is huge, is not news to anyone familiar with quantum mechanics or quantum computing, but the perspective presented here is quite novel. The key notion is the *relative* complexity of two quantum states, say $|A\rangle$ and $|B\rangle$: this is the minimum number of elementary unitary gates needed to go from $|A\rangle$ to $|B\rangle$. The state $|A\rangle$ evolves to $|B\rangle$ via elementary (local) unitary operators, and thus a computation on K qubits traces out a path in $\mathrm{SU}(2^K)$; therefore one may also compute the relative complexity of unitaries in the $\mathrm{SU}(2^K)$ manifold. The size of that space of *operators* (again, and henceforth, on K qubits) scales like 4^K (up to a slowly growing factor related to a "cutoff parameter," introduced to discretize the group). The hugeness of that space guarantees that for long periods of time in a quantum computation, the possible paths that computations take are exceedingly unlikely to "collide." Looking at the set of possible paths in the group space thus yields a tree, rooted at the identity, reminiscent of the Cayley graph of a free group. Ultimately, however, the paths *are* going to collide, and the result is in fact an expander graph. At this stage, which occurs at time exponential in K, the states reach maximal complexity, so the complexity ceases to grow. Incidentally, it is important for the sequel to clarify that here "time" simply means the number of steps of a quantum computation, what is referred to in the book as "clock time." Furthermore, each such step entails elementary operations that are assumed to connect k qubits, and are called "k-local operations," and which together involve all K qubits, an assumption here called "all-to-all."

The upshot of this analysis is that complexity, very much like entropy, keeps growing until it reaches a maximum. At that point, we reach "complexity equilibrium," which is analogous to a state of maximal entropy or thermal equilibrium. Indeed, it is exactly a state of maximum entropy if one thinks of the $\mathrm{SU}(2^K)$ space as a physical system, where each point in that space is regarded as a system of (fictitious) particles. It is, however, not the physical system of K qubits we are dealing with, and so Susskind calls it the "auxiliary system." The second law of thermodynamics as applied to that auxiliary system is called here the "Second Law of Quantum Complexity." After doubly exponential time in K, complexity can start to decrease and, as in a Poincaré recurrence, the cycle can start over. In his lecture, Susskind states that the main takeaway from this first part is, "Complexity is the entropy of an auxiliary system."

2 Black Holes

Part II turns to black holes *per se*. The motivation is a long-standing (but not very well studied) puzzle regarding black hole dynamics. It is well-known that when the event horizon of a black hole is formed, it continues to grow until it reaches maximum entropy, and thus thermal equilibrium. (That black holes *have* an entropy, which is proportional to the area of the horizon, was established in the 1970s by Bekenstein and Hawking.) However, the *interior* of the black hole continues to grow long past the time at which the horizon reaches thermal equilibrium. Since what is growing can't be entropy, what is it that is growing? Spoiler alert (although easy to guess, given the book's title): The answer is complexity.

To explain more technically how this is formulated, note that the argument presented here is largely in the context of AdS/CFT. This is the duality of Anti-de Sitter Space ("AdS") and Conformal Field Theory ("CFT"), conjectured by Maldacena in 1998 [Ma], as yet unproven but supported by much subsequent theoretical research. The discussion also largely applies to wormholes, also known as Einstein-Rosen (ER) bridges – conjoined black holes connecting possibly distant points in spacetime. An AdS wormhole is dual to the state in CFT that is known as the "Thermofield Double" ("TFD"). The big question is, as the throat of the wormhole is increasing, what is the corresponding thing that is increasing in the TFD state? The answer, argued by analogy here, is relative complexity. It is this wormhole/TFD duality that originally led to the conjecture that wormholes and quantum entanglement are really the same phenomenon (encapsulated in the symbolic equation "ER = EPR," hypothesized by Maldacena and Susskind [MS], where here EPR stands for Einstein, Podolsky, and Rosen, who first pointed out the phenomenon of entanglement[4]).

The main goal of Part II is to understand the connection between complexity and black holes. While this is not a rigorously proved connection, it is compellingly argued by examining the common behaviors of quantum computations as just outlined on the one hand, and properties of black holes on the other. The corresondence asserts that a black hole of entropy S corresponds to a circuit with S qubits (so "S" and the "K" of the preceding part are identified), that k-local gates correspond to k-local Hamiltonians, and clock time corresponds to "Rindler Time." Suffice it to say here that Rindler Time is the time component of a coordinate system for the exterior of a black hole, which can be well approximated locally by flat Minkowski space in the vicinity of the horizon. To see the connection between these two definitions of time, it turns out that the rate of change of the interior volume has the same behavior as the rate of change of the complexity of the circuit of k-local gates, assuming we identify clock time with Rindler time. These connections can be distilled into the equation $\mathcal{C} = \frac{V}{G\ell_{ads}}$, where V is the wormhole volume, G is the gravitational constant, and ℓ_{ads} is the radius of curvature of AdS. Susskind calls this the "complexity-volume (or CV) duality." Further support for CV duality is obtained by comparing the perturbation of a quantum circuit by a single qubit in the past (via a "precursor operator"), and analogously by the introduction of a thermal photon to a black hole in the distant past. The complexity of a quantum circuit for the precursor operator agrees remarkably well with formulae derived from known expressions for the growth of the interior volume of an AdS black hole.

Contrary to the scenario sketched at the beginning of this review, there is an evident paradox that suggests that crossing the event horizon would be an immediately fatal experience. The reason, it has been conjectured, is that just past the horizon an infalling observer would encounter a so-called "firewall," an intense high-energy shockwave, presumably created in the remote (exponential) past by the addition of a thermal photon, which blue shifts over time, gaining exponentially higher energy. It was argued by Almheiri, Marolf, Polchinski and Sully in 2013 [AMPS] that this is likely because a "typical" state of a black hole exhibits this behavior. In that argument "typical" considers all possible states, in the Haar measure. However, as explained here, a similar argument would reach the absurd conclusion that every one of us is in danger of being "burnt to a crisp in the next nanosecond," unless we take our the cosmological circumstances into account, namely, how we got to be where we are at the moment. Thus if one considers a typical *naturally occurring* black hole (e.g., one that resulted from stellar collapse), it is quite exceptional with respect to the Haar measure, and the creation of a firewall would require a highly unstable state of *decreasing* complexity in the remote past. "Firewalls are fragile," as one of the section titles of Chapter 17 states. While possible, it is exceedingly unlikely that they occur. (The material on firewalls concludes Part II of the book, but appears at the beginning of Lecture III in the live lectures.)

3 The Second Law of Complexity

Part III delves further into this analog of the second law of thermodynamics. Beginning with the latter, a key idea introduced by Schrödinger is that of "negentropy," which is the amount of entropy by which a system can increase. It is simply the maximum entropy minus the entropy of the system. By the classical laws of thermodynamics, it represents the amount of physical work that the system can perform. The analogous quantity for complexity is the difference between maximal complexity and complexity, which Susskind terms "uncomplexity." It is a measure of the amount of *computational* work a system can perform.

[4]Some of these ideas made a cameo appearance in my previous review on these pages (SIGACT News **52**(3), pp. 6–9), of Avi Wigderson's "Mathematics and Computation".

One can create negentropy (and hence the ability to perform more work) by, for example, taking two boxes containing gases, with identical (completely correlated) microstates. While the maximal entropy is doubled, the total entropy doesn't change. Thus the resultant negentropy equals the original entropy. In the quantum setting, a similar effect can be obtained by the simple expedient of adding a single qubit to a maximally entangled state. This mechanism is illustrated by a quantum algorithm for computing the trace of an operator in $\mathrm{SU}(2^K)$, using K qubits and one "clean qubit," based on work of Knill and Laflamme [KLf]. Via a simple quantum circuit, the clean qubit is entangled with the operator, and thus is no longer clean, but observations of that extra qubit yield enough information to obtain good approximations to the trace. Thus adding a single clean qubit to K qubits buys an uncomplexity of 2^K.

What does this have to do with black holes? The answer is remarkable: It has a geometric and physical interpretation. A "Wheeler-deWitt (WDW) Patch" is (roughly) a region of space-time bounded by the future and past light cones of an observer (let's say our astronaut Alice) exterior to a black hole. It turns out that the complexity is proportional to the Einstein-Hilbert action of the WDW patch; this is known as the "complexity-action correspondence." Since the complexity of the black hole is (up to a constant of proportionality) equal to the volume *behind* the horizon, it is proportional to the contribution to the action from behind the horizon. However, the *maximal* complexity equals the entire volume behind the horizon, and hence there is a portion of spacetime outside Alice's forward light cone, and inside the horizon, which is proportional to the uncomplexity. That is the portion of spacetime available to Alice. By the foregoing argument, a single clean qubit can increase the uncomplexity by an exponential amount. The upshot is that Alice can avoid encountering a firewall by sending only a thermal photon into the black hole. A few more wouldn't hurt.

4 Opinion

The above was an attempt (modestly successful, I hope) to convey the narrative that this book tells in a clear and coherent way. Full disclosure that I am no expert in this area, and my reading was a "hybrid" one. I first skimmed each part, then watched the corresponding lecture [Su], with the book close at hand, taking notes, and finally read that part with care. Threaded throughout this process were various excursions to sundry textbooks to refresh my memory on GR (which I first learned nearly 50 years ago, and had an opportunity to revisit this past January), and turning to various parts of the literature to bring me up-to-date on the recent research literature.

This book serves as an introduction to a vital and ongoing research program; as such, it would be of greatest interest to graduate students and researchers. Although much of it assumes an acquaintance with current literature in physics, a considerable portion of it is written and can therefore also be read from a more purely computational perspective. There too, an acquaintance with quantum computing would be helpful, but the quantum computing aspects that it draws from range from the elementary to the generally well-known (to quantum computer scientists), and touching on some very intriguing aspects of quantum complexity. One notable idea is that of relative complexity. This notion seems to attain greater depth when studied in conjunction with Riemannian geometry and its relation to quantum circuit complexity, pioneered by Nielsen [Ni] and others (e.g., [NDGD]) in the mid aughts. In that work, one first observes (as is done here) that a quantum computation traces out a path in the $\mathrm{SU}(2^K)$ group manifold. Based on a Hamiltonian representing the unitary operation of a quantum circuit, one can then define a right-invariant metric and thereby a Riemannian geometry on the space of operators. The lengths of minimal geodesics in that manifold impose lower bounds on circuit size. More generally, this ascribes a geometric meaning to relative complexity, as Susskind mentions a few times in the course of the book. It seems that further research in this area would be worthwhile, and may be given further impetus by the interaction between complexity and physics as expounded here.

In summary, this book lies at the intersection of three endlessly intriguing and puzzling problems: quantum complexity, black holes, and the second law of thermodynamics (as manifested here as the second law of complexity). Susskind is one of the principles in this research, and also a masterful, clear, and entertaining expositor. Read this book! And check out the lectures too [Su].

References

[AMPS] A. ALMHEIRI, D. MAROLF, J. POLCHINSKI, AND J. SULLY, "Black holes: complementarity or firewalls?" *Journal of High Energy Physics* **2013** 62 (2013).

[KLf] E. KNILL AND R. LAFLAMME, "Power of One Bit of Quantum Information," *Phys. Rev. Lett.* **81**(25) (1998), pp. 5672–5675.

[Ma] J. MALDACENA, "The large-N limit of superconformal field theories and supergravity," *International journal of theoretical physics* **38**(4) (1999), pp. 1113–1133

[MS] J. MALDACENA AND L. SUSSKIND, "Cool horizons for entangled black holes," *Fortsch. Phys.* **61**(9) (2013), pp. 781–811.

[Ni] M. A. NIELSEN, "A Geometric Approach to Quantum Circuit Lower Bounds," *Quantum Information and Computation* **6** (2006), pp. 213–262.

[NDGD] M. A. NIELSEN, M. R. DOWLING, M. GU, AND A. C. DOHERTY, "Quantum computation as geometry," *Science* **311** (2006), pp. 1133–1135.

[Su] L. SUSSKIND, Lectures at PiTP 2018:
- Lecture I: https://youtu.be/6OXdhV5BOcY
- Lecture II: https://youtu.be/EA_H0X9oMb4
- Lecture III: https://youtu.be/wjgkQG77pcc

Review of[5]
A Short Course in Computational Geometry and Topology
by Herbert Edelsbrunner
Springer, 2014
110 pages, Softcover, $69.99

Review by
Abdulai Gassama (agassama@clarku.edu)
Frederic Green (fgreen@clarku.edu)
Departments of Mathematics and Computer Science
Clark University, Worcester, MA

1 Overview

Computational geometry and topology are huge branches of mathematics. Focussing on concepts that lead to computation is one strategy to provide a concrete conceptual basis for ideas that hold in a more general context. Indeed, this short book gives an introduction to a surprisingly broad range of ideas that can serve as a good introduction to geometry and topology (even broadly conceived) for undergraduates.

2 Summary of Contents

After the following introductory chapter, the book is divided into four parts, each consisting of a number of chapters.

1. **Roots of Geometry and Topology**

   The book begins with a layman's introduction to face vectors, Platonic solids, convex polyhedra, and demonstrates these terms with various regular polytopes. One example of a convex polyhedron for one of the five Platonic solids is the face vector of an octahedron being the reverse of that of a cube, and using this duality to map out the vertices, edges, and faces of the octahedron bijectively to the faces. Only a small introduction to functional composition is required to understand this.

   From there, Edelsbrunner goes on to demonstrate how vertices, edges, and faces are all related to each other via the Euler formula, and how that represents the properties of the bounded convex polyhedra. This in turn leads to its generalization, the Euler-Poincaré characteristic.

   With the fundamentals layed out, this chapter moves on to further geometric perspectives, such as packing densities in two and then three dimensions.

   PART I, Tessellations:

2. **Voronoi and Delaunay Diagrams**

   The notion of convexity leads to a discussion of half-planes and, from there, to Voronoi diagrams in the plane. Given a finite set S, these are defined as a partition of the plane into regions, each of which consists of points closer to a point in S than any other point in S. Since such a "Voronoi region" is an intersection of half-planes, it is guaranteed to be convex. A finite set S can also be used to define the

Delaunay triangulation, which is the dual of the Voronoi diagram. Since the latter lies in the plane, the Delaunay triangulation is a planar graph. Via the Euler relation, and relating the graph to maximal planar graphs, this can be used to derive upper bounds on the number of edges and vertices in a Voronoi diagram. The chapter ends with an algorithm for the Voronoi diagram, which proceeds by adding triangles one at a time to the Delaunay triangulation, followed by an estimate of its run time. The chapter is succinct, with the reader only needing a basic knowledge of set theory and geometry to understand what's going on.

3. **Weighted Diagrams**

   This chapter generalizes the notion of distance to "weighted distance," which assigns weights to sites. Two schemes are discussed. The first simply subtracts the site's weight from the Euclidean distance (in a sense giving the site a nonzero size). This leads to Apollonius diagrams, whose regions are not necessarily convex. Subtracting the weight from the *square* of the Euclidean distance yields "power diagrams" and generalizations called *weighted* Voronoi and Delaunay diagrams; here the regions *are* convex. Some linear algebraic aspects of algorithms for obtaining geometric primitives for such diagrams are also discussed.

4. **Three Dimensions**

   After a quick introduction to lattices, this chapter illustrates three-dimensional constructions of Voronoi diagrams and Delaunay triangulations for the cubic, body-centered cubic, and face-centered cubic lattices. The visuals for the different $\mathbb{R}^3$ lattices help clarify the reader's understanding, alongside the formal definitions.

PART II, Complexes:

5. **Alpha Complexes**

   The idea of the "shape" of a discrete set of points is studied here. The complex hull is too coarse to get an intuitively satisfying notion of shape. A more refined idea is based on the α-hull, which is the basis of the notion of α-shape. The union of disks combined with the Voronoi diagram is used to construct the "alpha-complex," which is the Delaunay triangulation dual to the Voronoi diagram. Here α is the radius of closed disks centered at the sites. By varying α, we obtain a sequence of increasing complexes called a "filtration." This is instrumental in later chapters, especially in the treatment of persistent homology. The chapter ends with a discussion of the application of alpha-complexes to space-filling models of proteins.

6. **Holes**

   Alpha-complexes enable the classification of different types of holes, which are investigated in this relatively short chapter. Holes can be "voids" (e.g., the inner part of an annulus) or "pockets" (indentations). Filtrations can be used via thickening to identify pockets in two and three dimensions, and "tunnels" in three. Such techniques can be applied to branched peptides in two dimensions and, in three dimensions, to protein structure.

7. **Area Formulas**

   Space-filling models are common for simulating protein folding and the geometric characteristics of other biomolecules. Measuring areas is an important ingredient of this process, and is discussed in this chapter. Alpha-complexes can be used to simplify the area computation (based on the principle of inclusion-exclusion) for such space-filling models. The three pages in this chapter give the reader with a minimal understanding of set theory a good explanation of the algorithms.

PART III, Homology:

8. **Topological Spaces**

   Shifting from a geometric to a topological focus, this chapter introduces topological spaces, 2-manifolds, and compact surfaces. There is an appealing reprise, and extension, of Chapter 1 in the classification of surfaces via the Euler characteristic, and of Chapter 2 in the introduction of simplicial complexes as collections of convex regions (simplices), and triangulations.

9. **Homology Groups**

   This chapter begins with a fun metaphor to motivate the notion of homology. This "little creatures" metaphor serves to motivate p-chains and chain complexes. The essential definitions (p-simplices, boundary homomorphisms, p-cycles) as well as the Fundamental Lemma (that "the boundary of a boundary is zero"), serve well to establish the definition of homology groups and Betti numbers, promised in Chapter 1. Some calculations of particular Betti numbers are given. The chapter provides a nice introduction to the Euler-Poncaré Formula and to related topological computations, in a surprisingly compact and cogent manner.

10. **Complex Construction**

    This chapter goes further than the previous one to allow the construction of more general topological spaces. This is achieved first via abstract simplicial complexes, their geometric realization (along with a succinct statement of the Geometric Realization Theorem), homotopy, and homotopy equivalence. The chapter introduces the notion of a *nerve*, and the "Nerve Theorem," which gives a sufficient condition for two spaces to have the same homotopy type, and thus to more flexibility in computing such quantities as Betti numbers. A pleasing connection with previous chapters is that the Delaunay triangulation is (roughly) the nerve of the Voronoi diagram, and that an alpha complex has the same homotopy type as the set of balls it represents.

PART IV, Persistence:

11. **Filtrations**

    These were first introduced in Chapter 5 as an increasing sequence of α-complexes and are reviewed here. The computation of the Betti numbers of each complex can be achieved incrementally by a very fast algorithm. However, it remains a difficult problem to identify (out of the ensuing "topological noise) higher dimensional structures such as tunnels, and for this the idea of persistent homology is introduced. Persistent Betti numbers count the number of p-dimensional holes that "persist" for a certain interval in the filtration. The "barcode" of a filtration displays the sequence of "births" and "deaths" of classes in the sequence of homologies, and hence enable the identification of classes (and hence holes of various dimensions) that persist in the filtration.

12. **PL Functions**

    Piecewise linear ("PL") interpolation yields computationally convenient filtrations, introduced in this chapter. The theory of these "lower star filtrations" is discussed here, which (among other fundamental concepts) touches on the Morse inequalities. Particularly important is the statement of the Stability

Theorem for persistence diagrams (which give representations of complicated barcodes, introduced at the end of the previous chapter), which gives a sufficient condition for such diagrams to be stable under small perturbations in PL functions.

13. **Matrix Reduction**

    Representing boundary maps of a complex by binary matrices, this chapter shows how reducing matrices to normal form leads to alternative (and more general) algorithms for Betti numbers and persistence diagrams. Good algorithms for the latter require some modifications to the classical algorithms for normal forms. The chapter gives a concrete example, showing how to translate a reduced matrix to barcode.

3 Opinion

Choosing a concrete and accessible starting point, i.e., Voronoi diagrams and Delaunay triangulations, already engages the reader in important and subtle ideas. Throughout the book the author does a great job guiding the reader through topics stemming from these ideas. Thus the first few pages segue from Platonic solids to Euler's formula to the concept of convexity. There is a natural flow from there to generalizations such as weighted diagrams, complexes, and from there to homology, filtrations, and persistence. For another (more local) example, it is quite satisfying to see, in Chapter 10, the generalization of concrete simplicial complexes to abstract ones, going from there to geometric realizations, to homotopy types, thence to nerves, and then tying that back to Voronoi diagrams. The narrative is masterfully crafted from beginning to end.

Edelsbrunner is ever mindful to set forth the core motivations of each chapter before diving into their contents in depth, and where possible, to use appropriate metaphors to make the abstractions more understandable. A special feature of the book is the inclusion of a number of well-motivated applications, largely from molecular biology, for which the mathematical and algorithmic techniques are particularly well-suited. It should also be mentioned that at the end of each part there is a small set of interesting exercises to give the reader practice in applying the mathematics, and there are frequent pointers to the literature for readers who want to learn more.

All of these features make for a pedagogically sound and effective treatment. The text is certainly suitable for advanced undergraduates in mathematics. The principle prerequisites would be some basic acquaintance with set theory and linear algebra. While it may be challenging for some students, anyone would be well-advised to heed the author's advice to read the book "slowly, pay close attention to detail…and spend time to digest the material."

Both reviewers greatly enjoyed reading this excellent book. It is not just for the undergraduate (as one of us is), but also for the seasoned researcher (as is the other one) in an adjacent field desiring a quick but very clear introduction to this fascinating area. Highly recommended.

Review of[6]
The Age of Algorithms
by Serge Abiteboul and Gilles Dowek
Cambridge University Press, 2020
160 pages, Paperback, $19.99

Review by
S.V.Nagaraj svnagaraj@acm.org
VIT, Chennai Campus, India

1 Introduction

This book is about algorithms and their enormous influence on people in the current age. Algorithms are precise sets of rules to solve problems. They are ubiquitous and have a great effect on the lives of contemporary people, primarily due to technological advancement. A common example is searching for information using an Internet search engine such as Google. This book is not technical in nature and was first published in French as Le Temps des Algorithmes, by Éditions Le Pommier in 2020. The authors Serge Abiteboul and Gilles Dowek are computer scientists, with Institut National de Recherche en Informatique et en Automatique (INRIA), Rocquencourt and École Normale Supérieure, Paris respectively. They believe that algorithms have made life easier, nevertheless, they dread that algorithms may subjugate humans. This book is intended to serve as an eye-opener on the impact of algorithms on daily life.

2 Summary

This short book discusses what algorithms are; what they can accomplish; what they can't accomplish; their impact on work, employment, property; and questions such as whether they can be intelligent and whether they can have feelings, besides other topics. The book is not divided into chapters, rather twenty short sections.

The authors begin by saying that though algorithms intrigue or fascinate, they also disturb. They allege that algorithms are destroying jobs. Accident victims get compensation that is decided by algorithms, stock market crashes are attributed to trading algorithms, we are being spied upon by government algorithms, and humans lose to algorithms which excel in games such as chess and Go. They conclude the section by inferring that algorithms do not by themselves have any intention, and they are primarily what we want them to be. In the second section, the authors describe what an algorithm is, decribe the connections between algorithms and mathematics, offer an intro to algorithmic techniques such as divide and conquer, the brute force approach, greedy algorithms, and randomization. There is a brief look at machine learning too. The third section is on algorithms, computers, and programs. There is an interesting discussion about the first machines implementing algorithms, starting with the bells of cathedrals. What we think of as a phone in our pocket is actually a computer capable of functioning as a telephone, camera, watch, music player, and so on. The authors then introduce programming languages and explain the differences between programs and algorithms. The power of computers is illustrated by their ability to apply algorithms not only to symbolic information such as texts but also to digitized information such as images. The fourth section is on what algorithms (and computers) do. Computation, information management, communication, exploration, data analysis, signal processing, object control, manufacturing, modeling and simulation are illustrated as examples. The fifth section focuses on what algorithms cannot do. The limits of computation, too much computation time, constraints imposed by memory and heat; the impact of errors in algorithms, and bugs in software impose barriers on what algorithms can do. The authors state that a final limit is due to the difficulty of dialogue between a human and an algorithm. It is said that the plane crash of Air France Flight 447 from Rio to Paris probably happened due to the human pilots who did not correctly interpret the data presented by the computers.

The sixth section is on computational thinking. The pessimistic seventh section is on the end of employment. The authors state that perhaps the management of work time could benefit the most from low-cost information processing. The lower costs of information processing in turn allow for a better use of the resources, such as a car or a pencil sharpener. This could spell the end of employment and its replacement by a onetime contract. They feel that the model of salaried, and even hourly, employees is on its way out. The eighth section is on the end of work. The authors believe that in the age of algorithms, much less work may be required to provide the same goods or services as before. Algorithms for driving a car or translating a text may render drivers and translators obsolete. The washing machine, for example, was responsible for the disappearance of the washerwoman. The authors feel that attorneys, physicians, teachers, and other intellectual workers will also soon be partially replaced by algorithms. Increase in productivity will not necessarily result in idleness. For example, increase in agricultural productivity in the USA caused some farmers to move to other professions. The authors feel that the end of work disrupts distribution of wealth between the providers of capital and the providers of work. The ninth section is on the end of property. The age of algorithms is indeed the age of sharing free digital resources, for example, open-access archives of medical papers such as PubMed Central. The weakening of the concept of property leads us to an interesting question: "How can inventors, computer scientists, musicians, and others make a living if they abandon all ownership of the objects that they produce and all income linked to this ownership?" The authors make a nice observation: "In the age of algorithms, competitive balance is replaced by another law, winner takes all, until it is overthrown by another, younger, and more innovative winner." The tenth section is on governance in the age of algorithms. It looks at digital government, citizen participation in democracy. The authors rightly point out that though a wealth of information is available to the public, "the deluge of information, however, complicates the selection of pertinent information by individuals who can also be manipulated by disinformation campaigns."

The eleventh section is on community algorithms. The authors state "algorithms are often perceived as the root of all evil, responsible for the disappearance of jobs, restriction of civil liberties, and the dehumanization of the world, among other things." They pose many interesting questions: "Which decisions should be relegated to algorithms? How do we challenge decisions made by algorithms? Can algorithms be considered morally and legally responsible for their actions?... But can we, in the age of algorithms, go even further and entrust criminal convictions or the parole of convicted criminals to such algorithms?" The authors look at the possibility of challenging the decisions made by algorithms. The twelfth section is on the reponsibility of algorithms. The question posed here by the authors is whether an algorithm can do harm? The answer is obvious. The authors state "the same big data analytics algorithm can make it possible for physicians to personalize treatment and save human lives, and for governments to spy on their citizens in disregard of privacy rights. ...Many algorithms – sometimes thousands – interact, exchange data, reason, suggest, and decide for us. Three examples will serve to illustrate this: self-driving cars, digital personal assistants (such as Siri or Google Now), and the purchase and sale of financial products." Acording to the authors, automated trading serves as a good example for a discussion about the question of algorithm responsibility. They strongly believe that "algorithms can be a force for good as well as bad, and that we should use algorithms only when we are confident that they will perform as intended. We cannot allow self-driving vehicles to turn our roads into a jungle." The thirteenth section is on personal data and privacy. "Thanks to computers and learning algorithms, countries can now run massive surveillance operations of the entire population." The authors rightly point out that governments "aren't the only ones interested in our data. Corporations, especially the key actors of the web, also understand the value of personal data. Will internet users consent to be continuously monitored and analyzed?" An example illustrating the drawbacks of such monitoring is that of a teenage girl whose father learned of her pregnancy in 2012 through a targeted ad. "By analyzing the young woman's purchases, the algorithms used by the company Target correctly determined that she was probably pregnant and sent her discount coupons for baby clothes and cradles." The authors feel that "health data should not be locked up, but we must decide who can access such data and, more importantly, what they can do with it. We must also develop the necessary techniques for analyzing massive amounts of data while preserving confidentiality." The fourteenth section is on fairness, transparency, and diversity. We have certain expectations of the algorithms we use. For example, we would like them to be fair. In order to prove that an algorithm is actually biased, we need to know how it works. This brings us to the issue of algorithmic transparency. The authors point out that "lack of transparency in a number of algorithmic decisions, from e-commerce pricing to judicial sentencing, has been shown to reinforce biases based, for instance, on race or gender." They conclude that "algorithms are not intrinsically fair or transparent. Nor are they unfair or opaque. They are exactly as we make them." The fifteenth section is on computer science and ecology. Discussions on algorithms and global warming, algorithms and complex systems, computer science – the consumer of electricity all make for interesting reading.

The sixteenth section is on computer science and education. Today it is really impossible to understand our world without some grasp of computer science. This is why, as the authors state, "it has become necessary to also teach computer science alongside subjects like physics and biology." A good question posed by the authors is: "Are there professions in which computer science doesn't play a role?" They also give advice on teaching: "Instead of teaching the latest programming language, we should teach the principles of programming, the fundamental structures common to all languages..." The authors feel that the best results in computer science instruction are obtained though project-based learning, mostly carried out in teams. The seventeenth section is on the augmented human. The eighteenth section asks a common question: can an algorithm be intelligent? This question brings up two others: "What does the adjective intelligent mean? Can we create an intelligent being?" The authors opine that "neither Minsky nor Turing was able to truly clarify the notion of intelligence, and we still do not have a satisfactory answer to our initial question: What is intelligence?" The nineteenth section asks another interesting question: can an algorithm have feelings? The last section is titled "Time to choose." The authors conclude that "with algorithms, Homo Sapiens has finally created a tool equal to their aspirations, a tool that makes it possible to build a world that is better, freer, and fairer. The choice is ours."

3 Opinion

This short and interesting book provides a non-technical introduction to the age of algorithms. The growing influence of algorithms on our daily lives is well portrayed by the authors. Despite the fears of the authors, not everything can be done by machines and algorithms. Algorithms are essentially what we want them to do. The book is worth reading many times even by those unfamiliar with algorithms or computer science.