



Final Exam

Name: (print neatly) _____

Instructor: _____

(sign) _____

1. (20 pts) Find the following limits. If a limit fails to exist, then briefly explain why.

a) $\lim_{x \rightarrow 0} \sin(4x) - x$

b. $\lim_{x \rightarrow \infty} \frac{4x^2 - 5}{3x^2 + 4x + 2}$

c. $\lim_{x \rightarrow 0} \frac{(2x + 3) \sin(x)}{x}$

d. $\lim_{x \rightarrow 2} \frac{x + 2}{x - 2}$

2. (25 pts) Find the following derivatives.

a. $\frac{d}{dx} x \tan(x)$

b. $\frac{d}{dx} \left(\frac{\sqrt{x}}{x-1} \right)$

c. $\frac{d}{dx} \sin^3(2x - \pi)$

d. $\frac{d}{dx} \sqrt{1 - 3x^2}$

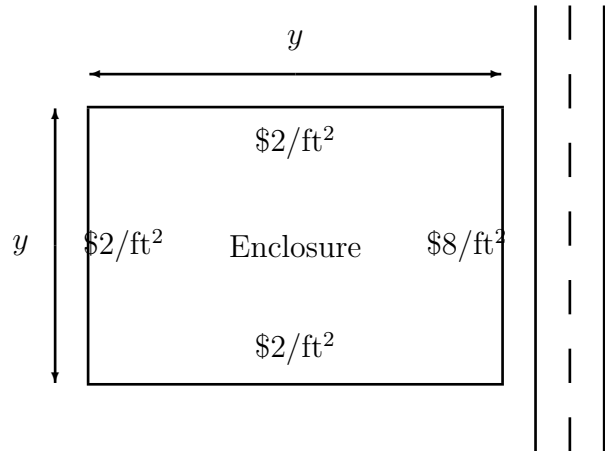
e. $\frac{d^2}{dx^2} (1 - x^3 + x^6)$

3. (**15 pts**) A 50 foot ladder is leaning against a tall building. If the base of the ladder is slipping away from the building at 3 inches per minute, how fast is the top of the ladder descending when it is 40 feet up from the ground?

4. (15 pts) A rectangular region is to be enclosed by fencing which costs \$8 per foot on the east side (facing a road) and \$2 per foot on the other three sides. The total budget for materials is \$1000.

Answer questions a-c below to determine the dimensions of the largest rectangle that can be enclosed.

a) Express the total cost in terms of x and y .



b) Express the quantity to be maximized.

c) Use appropriate techniques to find the dimensions of the largest rectangle that can be enclosed.

5. (25 pts) The function $f(x)$ and its derivatives are given:

$$\begin{aligned}f(x) &= \frac{x^2}{x^2 - 2x + 2} \\f'(x) &= \frac{-2x(x - 2)}{(x^2 - 2x + 2)^2} \\f''(x) &= \frac{4(x - 1)(x - 1 - \sqrt{3})(x - 1 + \sqrt{3})}{(x^2 - 2x + 2)^3}\end{aligned}$$

[Hint: The denominator is always positive.]

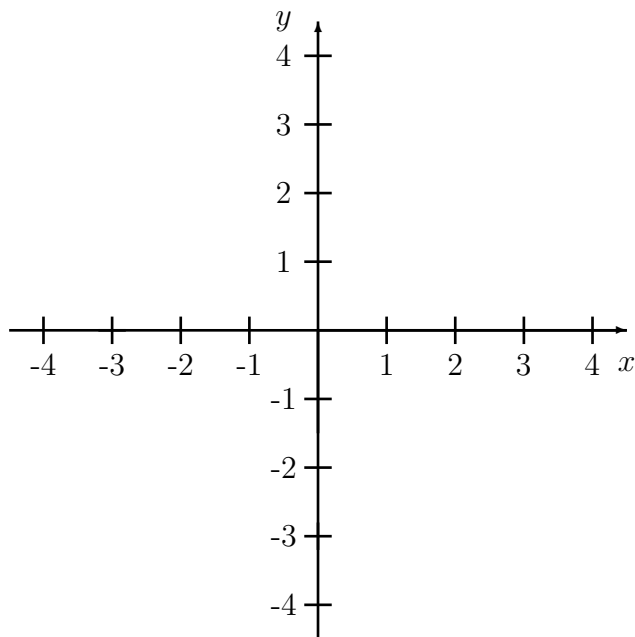
a) Find all intervals on which $f(x)$ is increasing, and those on which $f(x)$ is decreasing.

b) Find all critical values and determine whether each is a local max or a local min.

c) Find all inflection points.

d) Find all intervals on which $f(x)$ is concave up, and those on which $f(x)$ is concave down.

e) Give a neat sketch of $f(x)$ below labelling all critical and inflection points, and showing all asymptotes.



Prob	Pts
1	
2	
3	
4	
5	
Total	