

Test #1

Name: (print neatly) _____

Instructor: _____

(sign) _____

1. (**14 pts**) Determine whether each of the following functions is continuous at the given value. Give a brief explanation.

a) $g(x) = \frac{\cos(3x)}{x^2 - 4}$ at $x = 0$.

b) $f(x) = \begin{cases} x & \text{for } x < 0 \\ 1 & \text{for } x = 0 \\ \frac{\sin(x)}{x} & \text{for } x > 0 \end{cases}$ at $x = 0$.

2. (20 pts) If $\lim_{x \rightarrow 9} f(x) = 2$ and $\lim_{x \rightarrow 9} g(x) = 3$, evaluate all of the following limits which you know exist.

a) $\lim_{x \rightarrow 9} \frac{f(x)}{g(x)}$

b) $\lim_{x \rightarrow 9} \frac{f(x) - 2}{g(x) - 2}$

c) $\lim_{x \rightarrow 9} \frac{f(x) - 3}{g(x) - 3}$

d) $\lim_{x \rightarrow 9} xf(x)$

3. (6 pts) Suppose $f(x) = 1 + x^2 - x^5$.

Show that the equation $f(x) = -3$ has a solution on the interval $[0, 3]$.

[Hint: Use the Intermediate Value Theorem.]

4. (25 pts) Find the following limits. If a limit fails to exist, then briefly explain why.

a) $\lim_{x \rightarrow 1} [(3x^2 - 2)(5x - 4)(8x^3 - 6)]$

b) $\lim_{x \rightarrow 3} \frac{3x - x^2}{x^2 - 2x - 3}$

c) $\lim_{x \rightarrow 0} \frac{x}{x^2 - 1}$

d) $\lim_{x \rightarrow 0^-} \frac{-x}{|-x|}$

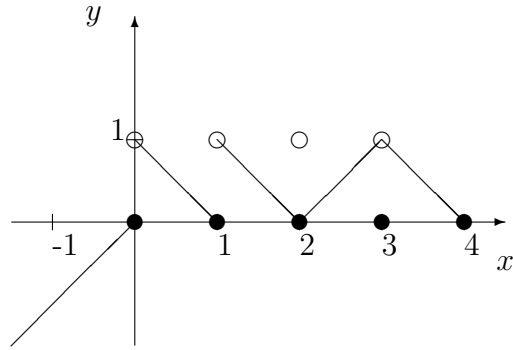
e) $\lim_{x \rightarrow 0} \frac{1 - x}{-x}$

5. (12 pts) Find the following limits. If a limit fails to exist, then briefly explain why.

a) $\lim_{x \rightarrow 0} \sin^2(\pi - x)$

b) $\lim_{x \rightarrow 0} \frac{\sin(3x)}{x}$

6. (15 pts) The function $f(x) = x$ has domain $x \leq 4$. For $x < 0$, $f(x) = x$ and for $0 \leq x \leq 4$ the graph is as shown.



a) What is the range of $f(x)$?

b) Compute $\lim_{x \rightarrow 1^-} f(x)$.

c) For what values of x is $f(x)$ continuous?

d) What values of x correspond to jump discontinuities?

e) What values of x correspond to removable discontinuities?

7. (8 pts) a. State carefully the ϵ - δ formulation of the definition of the limit, $\lim_{x \rightarrow a} f(x) = L$.

b. Prove via the ϵ - δ definition that $\lim_{x \rightarrow 2} 10x - 8 = 12$.

Prob	Pts
1	
2	
3	
4	
5	
6	
7	
Total	