CLARK UNIVERSITY

Name: ______________________________

Circle your instructor's name:

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Math 120 Calculus I

Final Exam

December 2016

This is a closed-book, closed-notes test. Calculators are not allowed. Please turn off your cellphone and any other electronic equipment during the test.

Leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Be sure that your proofs and computations are easy to read. Points for each problem are in square brackets.

1. [22] Consider the function $f(x) = \dfrac{x}{1+x^2}$. Its derivative is $f'(x) = \dfrac{1-x^2}{(1+x^2)^2}$, and its second derivative is $f''(x) = \dfrac{4x^3-6x}{(1+x^2)^3}$.

a. [3] What are the x-intercepts and y-intercepts of f?

b. [3] What are the critical points for f?

c. [3] What are the inflection points for f?

d. [4] Are there any vertical asymptotes? Are there any horizontal asymptotes

e. [6] Sketch the graph of f. Show intercepts, critical points, inflection points, and asymptotes.

2. [12; 6 points each part] On limits. Evaluate each of the following limits in parts a and b if it exists, but if it doesn't then explain why.

a. $\displaystyle\lim_{x\to 0} \frac{\sin^2 3x}{5x^2}$

b. $\displaystyle\lim_{x\to\infty} \sqrt{\frac{4x^3 - 2x}{9x^3 + 1}}$

c. Suppose that $f'(x) = \sqrt{x^2+1}$. Use that information to evaluate

$$\lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$$

3. [24; 6 points each part] Differentiation. Do not simplify your answers. Use parentheses properly.

a. For $f(x) = x \ln x$, find $f'(x)$.

b. Evaluate $\dfrac{d}{dx} \tan^3(2x - \pi)$.

c. Let $f(t) = \dfrac{e^t + t^{2/3}}{1 + \tan t}$. Find $f'(t)$.

d. Let $F(x) = \displaystyle\int_4^x \frac{t^t + \ln(t^2 + 1)}{1 + \sqrt{t}}\, dt$. Find $F'(x)$. (Hint: do not try to evaluate the integral.)

4. [10] Determine the function $f(x)$ whose derivative is $f'(x) = 6x^2 - 4x + 2$ and whose value at $x = 1$ is $f(1) = 9$.

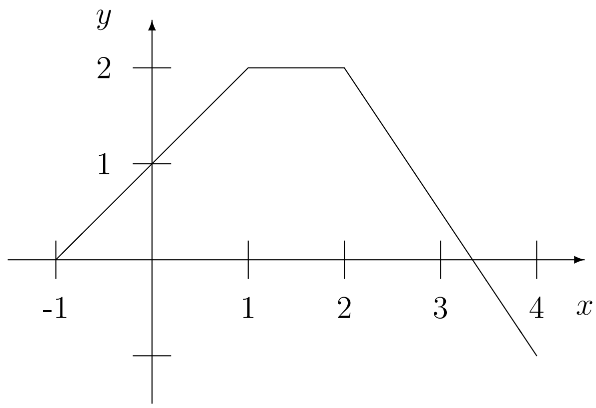
5. [10] A cylindrical aluminum can is to be constructed to have a volume of 1000 cubic cm. Let h denote the height of the can and r the radius of the base. Recall that the volume of a cylinder with height h and radius r of the base is $V = \pi r^2 h$, and the total surface area is $A = 2\pi r^2 + 2\pi rh$. Determine the dimensions of the cylinder to minimize the surface area A of the can. Your final answer should indicate the values of r and h.

6. [12; 6 points each part] Evaluate the following integrals. Note that the first one is an indefinite integral and the second one is a definite integral.

a. $\displaystyle\int (5e^x + 3\cos x)\, dx$

b. $\displaystyle\int_1^4 \left(x^2 + \frac{1}{2\sqrt{x}} \right) dx$

7. [10] The graph of a function $f(x)$ is drawn below. Its graph consists of three line segments.



Determine the value of the integral $\displaystyle\int_{-1}^{4} f(x)\,dx$.

#1.[22]	
#2.[12]	
#3.[24]	
#4.[10]	
#5.[10]	
#6.[12]	
#6.[10]	
Total	