CLARK UNIVERSITY

Name: _____

Circle your instructor's name:

Joyce Pendharkar Winders

Math 120 Calculus I

First Test

September 2016

This is a closed-book, closed-notes test. Calculators are not allowed. Please turn off your cellphone and any other electronic equipment during the test.

Leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Be sure that your proofs and computations are easy to read. Points for each problem are in square brackets.

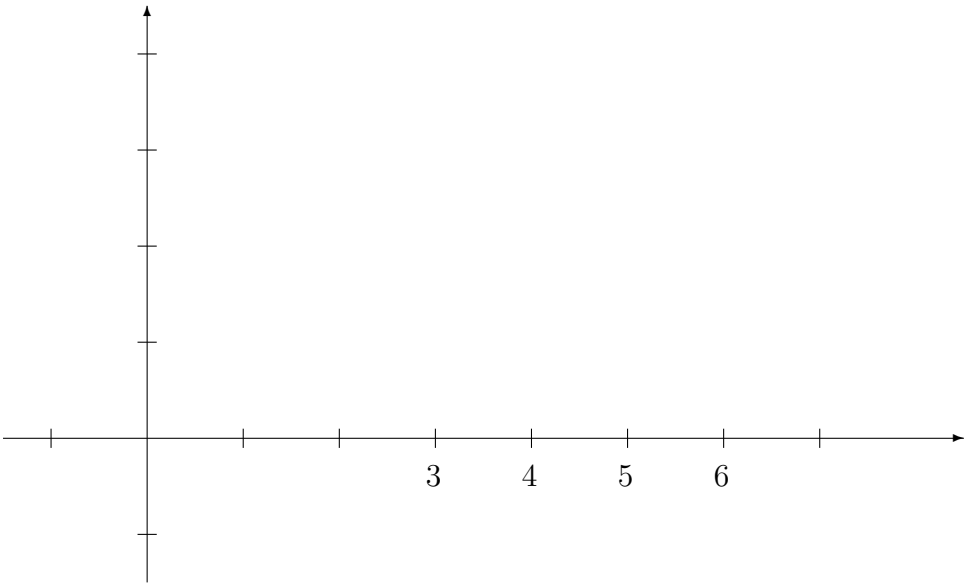
1. [12] On the intutitive concept of limit and continuity.

Sketch the graph $y = f(x)$ of a single function that is defined everywhere with the following three properties:

(1). $\lim_{x\to 3} f(x)$ does exist but does not equal $f(3)$,

(2). the left and right limits, $\lim_{x\to 4^-} f(x)$ and $\lim_{x\to 4^+} f(x)$, both exist, but are not equal, and

(3). the limit $\lim_{x\to 5} f(x)$ doesn't exist for some reason other than the left and right limits aren't equal (for example, either the left or right limit or both don't exist).



2. [12] On limits of average rates of change. Let $f(x) = x^2 - 3x$.

a. [8] Write down an expression that gives the average rate of change of this function over the interval between x and $x + h$, and simplifly the expression.

b. [4] Compute the limit as $h \to 0$ of your answer in part a.

3. [24; 8 points each part] Evaluate the following limits. If a limit diverges to $\pm\infty$ it is enough to say that it doesn't exist. (Use the properties of limits weve discussed so far in class. Later on this semester well discuss LHopitals rule, but dont use that to determine these limits.)

a. $\lim_{x\to 1} \dfrac{x^2-1}{x^2-3x+2}$

b. $\lim_{x\to 0} \dfrac{\dfrac{1}{x-1}+\dfrac{1}{x+1}}{x}$

c. $\lim_{x\to 0} \dfrac{4\sin x}{5x}$.

4. [15] On the formal definition of limit.

Consider the limit $\lim_{x\to 5}(2x-3)$ which, of course, has the value 7. Since it has the value 7, that means that for each $\epsilon > 0$, there exists some $\delta > 0$, such that for all x, if $0 < |x-5| < \delta$, then $|(2x-3)-7| < \epsilon$.

Let $\epsilon = \frac{1}{2}$. Find a value of δ that works for this ϵ. (Show your work.)

5. [10] Suppose that θ is an angle between $-\pi/2$ and 0, and that $\cos\theta = \frac{1}{2}\sqrt{2}$. Determine the value of $\sin\theta$.

6. [10] Write the expression $e^{\ln(9x)-\ln(5y)}$ without using either e or ln.

7. [18; 6 points each part] Suppose that $\lim_{x\to\pi} f(x) = 5$ and $\lim_{x\to\pi} g(x) = 3$. Evaluate each of the following limits, or explain why it doesn't exist

a. $\lim_{x\to\pi} \dfrac{f(x)}{g(x)}$

b. $\lim_{x\to\pi} \dfrac{f(x)}{g(x) + 3\cos x}$

c. $\lim_{x\to\pi} \sqrt{x + f(x)g(x)}$

#1.[12]	
#2.[12]	
#3.[24]	
#4.[15]	
#5.[10]	
#6.[10]	
#7.[18]	
Total	