Math 120 Calculus I

First Test Answers
September 2016

Scale. 95–101 A+. 89–94 A. 85–88 A-. 81–84 B+. 77–80 B. 73–76 B-. 67–72 C+. 61–66 C. 54–60 C-. 48–53 D+. 40–47 D. Median 75.

Extra credit. There will be an extra credit "challenge test" you can take on line available all day Saturday and Sunday, Oct 8–9, for 48 hours.

You'll find the challenge test will cover as the chapter 2 post test on MyMathLab. If you got less than 65 on the test you're required to take this challenge test, if above 65 it's optional. It includes topics from section 2.6 as well as the rest of that chapter. You'll be able to do each question on it only once. You may use a calculator if you need it, but no book, notes, or internet. Do the test entirely by yourself. Do not get help from anyone or talk to anyone about the test until Monday after the test ends.

Whatever questions you do correctly on the challenge test will increase your grade for the first test. You can get up to half the difference between your test grade and 100. So, for example, if you got 70 on this test, a C+, you could increase your grade by 15 to 85, an A-, by getting all 24 problems correct on the challenge test, but if only 12 correct, then your grade would increase by 7.5 to 77.5, a B.

1. [12] On the intutitive concept of limit and continuity.

Sketch the graph $y = f(x)$ of a single function that is defined everywhere with the following three properties:

(1). $\lim_{x\to 3} f(x)$ does exist but does not equal $f(3)$,

(2). the left and right limits, $\lim_{x\to 4^-} f(x)$ and $\lim_{x\to 4^+} f(x)$, both exist, but are not equal, and

(3). the limit $\lim_{x\to 5} f(x)$ doesn't exist for some reason other than the left and right limits aren't equal (for example, either the left or right limit or both don't exist).

The curve above $x = 3$ should have a point missing and repositioned directly above or below.

There should be a jump discontinuity in the curve above $x = 4$.

For the limit not to exist for $x = 5$ but not be a jump discontinuity like at $x = 4$, you could either draw an oscillating discontinuity or let the limit be $\pm\infty$.

Note that the graph of the function should extend to the left of $x = 3$ and to the right of $x = 5$ as well as between those two values.

2. [12] On limits of average rates of change. Let $f(x) = x^2 - 3x$.

a. [8] Write down an expression that gives the average rate of change of this function over the interval between x and $x + h$, and simplifly the expression.

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{((x+h)^2 - 3(x+h)) - (x^2-3x)}{h} \\
&= \frac{x^2+2xh+h^2-3x-3h-x^2+3x}{h} \\
&= \frac{2xh+h^2-3h}{h} \\
&= 2x+h-3
\end{aligned}$$

b. [4] Compute the limit as $h \to 0$ of your answer in part a.

$$\lim_{h\to 0}(2x+h-3) = 2x-3$$

3. [24; 8 points each part] Evaluate the following limits. If a limit diverges to $\pm\infty$ it is enough to say that it doesn't exist. (Use the properties of limits weve discussed so far in class. Later on this semester well discuss LHopitals rule, but dont use that to determine these limits.)

a. $\lim_{x\to 1} \dfrac{x^2-1}{x^2-3x+2}$

This is an algebraic function of the form 0/0, so you need to use algebra to simplify the expression before taking the limit.

$$\begin{aligned}
\lim_{x\to 1} \frac{x^2-1}{x^2-3x+2} &= \lim_{x\to 1} \frac{(x+1)(x-1)}{(x-2)(x-1)} \\
&= \lim_{x\to 1} \frac{x+1}{x-2} \\
&= \frac{2}{-1} = -2
\end{aligned}$$

b. $\lim_{x\to 0} \dfrac{\dfrac{1}{x-1}+\dfrac{1}{x+1}}{x}$

Again, this is an algebraic function of the form 0/0, so simplify before taking the limit.

$$\begin{aligned}
\lim_{x\to 0} \frac{\dfrac{1}{x-1}+\dfrac{1}{x+1}}{x} &= \lim_{x\to 0} \frac{1}{x}\,\frac{(x+1)+(x-1)}{(x-1)(x+1)} \\
&= \lim_{x\to 0} \frac{2x}{x(x-1)(x+1)} \\
&= \lim_{x\to 0} \frac{2}{(x-1)(x+1)} \\
&= \frac{2}{-1} = -2
\end{aligned}$$

c. $\lim_{x\to 0} \dfrac{4\sin x}{5x}$.

Recall that $\lim_{x\to 0} \dfrac{\sin x}{x} = 1$. Therefore the limit under question is $\frac{4}{5}$.

4. [15] On the formal definition of limit.

Consider the limit $\lim_{x\to 5}(2x-3)$ which, of course, has the value 7. Since it has the value 7, that means that for each $\epsilon > 0$, there exists some $\delta > 0$, such that for all x, if $0 < |x-5| < \delta$, then $|(2x-3)-7| < \epsilon$.

Let $\epsilon = \frac{1}{2}$. Find a value of δ that works for this ϵ. (Show your work.)

Simplify the goal $|(2x-3)-7| < \frac{1}{2}$. First $|2x-10| < \frac{1}{2}$. Then to make it match the given information $0 < |x-5| < \delta$, divide $|2x-10| < \frac{1}{2}$ by 2 to get $|x-5| < \frac{1}{4}$. Thus $\delta = \frac{1}{4}$ will do. (There are other algebraic steps that will lead to the same conclusion.)

Alternatively, you could sketch the graph to determine a value for δ.

5. [10] Suppose that θ is an angle between $-\pi/2$ and 0, and that $\cos\theta = \frac{1}{2}\sqrt{2}$. Determine the value of $\sin\theta$.

There are several ways to determine $\sin\theta$. Here's one. By the Pythagorean identity, $\sin^2\theta + \cos^2\theta = 1$, therefore $\sin^2\theta + \frac{1}{2} = 1$, so $\sin^2\theta = \frac{1}{2}$. Then $\sin\theta$ must be either $\sqrt{1/2} = \frac{1}{2}\sqrt{2}$ or its negation. Since $-\pi/2 \le \theta \le 0$, it's the negation. So $\sin\theta = -\frac{1}{2}\sqrt{2}$.

Alternatively, you could draw a unit circle and a right triangle in it.

6. [10] Write the expression $e^{\ln(9x)-\ln(5y)}$ without using either e or ln.

Using properties of exponentiation and logs,

$$e^{\ln(9x)-\ln(5y)} = \frac{e^{\ln(9x)}}{e^{\ln(5y)}} = \frac{9x}{5y}.$$

7. [18; 6 points each part] Suppose that $\lim_{x\to\pi} f(x) = 5$ and $\lim_{x\to\pi} g(x) = 3$. Evaluate each of the following limits, or explain why it doesn't exist

a. $\lim_{x\to\pi} \dfrac{f(x)}{g(x)}$

Since the limit of the quotient is equal to the quotient of the limit provided the denominator does not approach 0, therefore this limit is $\frac{5}{3}$.

b. $\lim_{x\to\pi} \dfrac{f(x)}{g(x) + 3\cos x}$

Note that $\cos x$ approaches $\cos\pi$ which equals -1. Therefore the denominator approaches 0. Since the numerator approaches 5, therefore the limit of the quotient does not exist.

c. $\lim_{x\to\pi} \sqrt{x + f(x)g(x)}$

The limit is $\sqrt{\pi + 15}$.