Name: _____

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Math 121 Calculus II

First Test

February 2014

You may use a calculator. Leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Points for each problem are in square brackets.

1. [20; 10 points each part] **Indefinite integrals and antiderivatives.**

a. A function f has the derivative $f'(x) = 6x^2 - 6x + 4$, and $f(2) = 3$. Determine the function f from that information.

b. Evaluate the indefinite integral

$$\int (\sec^2 x + \cos 3x)\, dx =$$

2. [20; 10 points each part] **On definite integrals.** Evaluate the following integrals. Show your work for credit. (You do not have to find the answer decimally; an unsimplified expression involving numbers is sufficient.)

a. $\displaystyle\int_0^{\pi/4} \sin^2 x \, \cos x \, dx$

b. $\displaystyle\int_3^5 \frac{1}{x+2}\, dx$

3. [20; 10 points each part] **On integrals and derivatives.** Let $f(x) = \displaystyle\int_2^x \frac{t^3+3}{t^2+1}\,dt$. Then $f'(3)$ equals (circle one)

A) 11/5 B) 6 C) 4/5
D) 1 E) 0 F) 3

b. In the first semester of calculus, you found derivatives of various functions like

$$f(x) = \left(\frac{3x}{x^2+1}\right)^4.$$

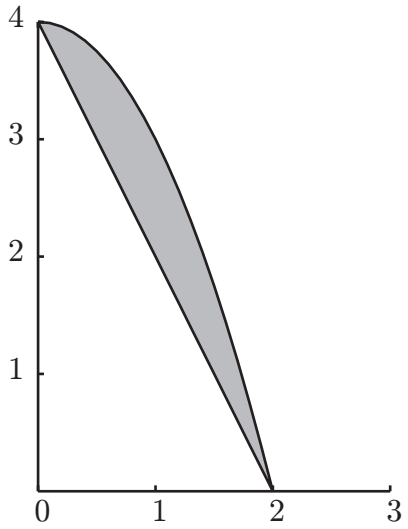
For that one, you used the chain rule and the quotient rule to find that

$$f'(x) = \frac{324x^3(1-x^2)}{(x^2+1)^5}.$$

Use that result to evaluate the following integral

$$\int_0^2 \frac{x^3(1-x^2)}{(x^2+1)^5}\,dx =$$

4. [10] **On areas between curves.** Write down an integral which gives area of the finite region bounded between $y = 4 - x^2$ and $y = 4 - 2x$ for $0 \leq x \leq 2$, sketched below. Do not compute the value of the integral.



5. [10] **On volumes of solids of revolution.** The region described in **4** is rotated about the x-axis to create a solid of revolution. Write down an integral which gives the volume of that solid. Do not compute the value of the integral.

6. [20; 10 points each part] **On arc lengths and surfaces of revolution.** Consider the curve given by the equation $y = \ln x$ between $x = 1$ and $x = e$.

a. Write down an integral which gives the length of this curve. Do not evaluate the integral.

b. That curve is rotated around the x-axis to form a surface of revolution. Write down an integral which gives the area of that surface. Do not compute the value of the integral.

#1.[20]	
#2.[20]	
#3.[20]	
#4.[10]	
#5.[10]	
#6.[20]	
Total	