CLARK UNIVERSITY

Name: ____________________________

Circle your instructor's name:

Broker Hill Joyce Pendharkar

Math 121 Calculus II

Final Exam

May 2015

This is a closed-book, closed-notes test. Calculators are not allowed. Please turn off your cellphone and any other electronic equipment during the test.

Leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Be sure that your proofs and computations are easy to read. Points for each problem are in square brackets.

1. [5] On volumes of solids of revolution. Consider the region below the curve $y = 8\sin 2x$ and above the x-axis for x between 0 and $\pi/2$. Rotate that region around the x-axis to get a solid of revolution. Write down an integral which gives the volume of that solid of revolution. Do not compute the value of the integral.

2. [6] On arc lengths. Suppose that a function f has the derivative $f'(x) = \ln x$. Consider the curve $y = f(x)$ for x in the interval $[2, 8]$. Write down an integral which gives the length of that portion of the curve. Do not compute the value of the integral.

3. [7] A common population model assumes the rate of change of a population is proportional to the population. Using that model answer the following question. (You can leave your answer as an expression.)

The biomass of a yeast culture is initially 12 grams. After 10 minutes the mass is 15 grams. How long will it take for the mass to double from its initial value?

4. [10; 5 points each part] On integration.

a. Evaluate the indefinite integral $\int xe^{-x}\,dx$

b. Use your answer in part **a** to determine whether the improper integral $\int_1^\infty xe^{-x}\,dx$ converges or diverges. If it converges, find its value.

5. [14; 7 points each part] Evaluate the following integrals. Note that one of them is a definite integral, the other an indefinite integral.

a. $\displaystyle\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{x^2-1}}\,dx$

b. $\displaystyle\int \frac{1}{x^2-3x+2}\,dx$

6. [14; 7 points each part] On limits of sequences. State whether the sequence converges or diverges. If it converges, find it's limit. If you use l'Hôpital's rule, point out where you use it.

a. $\lim_{n\to\infty} \frac{\sqrt{n^2+1}}{4n+9}$

b. $\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{2n}$

7. [5] Determine the sum of the series $\sum_{n=1}^{\infty} \frac{2^n}{3^n}$.

8. [18; 6 points each part] On convergence of series with positive terms. For each series, apply one or more convergence tests to determine whether the series converges or diverges. State which test(s) you use and explain your conclusion.

a. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln n}$

b. $\displaystyle\sum_{n=1}^{\infty} \frac{10^n}{n!}$

c. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$. Suggestion: integral test.

9. [15; 5 points each part] For each of the following two alternating series determine if it diverges, converges conditionally, or converges absolutely. Explain your conclusions.

a. $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \cdots$

b. $\frac{1!}{10^1} - \frac{2!}{10^2} + \frac{3!}{10^3} - \frac{4!}{10^4} + \cdots$

c. $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} - \cdots$

10. [6] For the function $f(x) = \arcsin x$, determine the third-order Taylor polynomial for f, that is, find a_0 through a_3 in the power series $a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$ that represents f. To save you time, here are the first few derivatives of $\arcsin x$.

$f'(x)$	$f''(x)$	$f'''(x)$	$f^{(4)}$
$\frac{1}{\sqrt{1-x^2}}$	$\frac{x}{(1-x^2)^{3/2}}$	$\frac{2x^2+1}{(1-x^2)^{5/2}}$	$\frac{6x^3+9x}{(1-x^2)^{7/2}}$

#1.[5]	
#2.[6]	
#3.[7]	
#4.[10]	
#5.[14]	
#6.[14]	
#7.[5]	
#8.[18]	
#9.[15]	
#10.[6]	
Total	