This is a closed-book, closed-notes test. Calculators are not allowed. Please turn off your cellphone and any other electronic equipment during the test.

Leave your answers as expressions such as $e^2 \sqrt{\frac{\sin^2(\pi/6)}{1 + \ln 10}}$ if you like. Show all your work for credit. Be sure that your proofs and computations are easy to read. Points for each problem are in square brackets.

1. [16] Evaluate the following indefinite integral.

$$\int \frac{\sqrt{x^2 - 4}}{x} \, dx$$
2. [24; 12 points each part] Evaluate the following indefinite integrals.

a. \( \int x^2 \sin x \, dx \)

b. \( \int \cos^3 x \sin^2 x \, dx \)
3. [12] Evaluate the following definite integral.

\[ \int_{1}^{e} x \ln x \, dx \]

4. [12] Solve the separable differential equation \( \frac{dy}{dx} = e^{-y} \cos x \). Your answer should express \( y \) as a function of \( x \).
5. [20] Consider the rational function \( \frac{7x + 7}{x^2 + 3x - 10} \).


b. [8] Write the rational function as a sum of partial fractions.

c. [8] Use what you found in part b to evaluate this integral.

\[
\int \frac{7x + 7}{x^2 + 3x - 10} \, dx
\]
6. [16; 8 points each part] The most common form of radium, radium-226, has a half life of 1601 years.

a. Write down a formula that gives the amount $y$ of radium left after a period of $t$ years when the initial amount is $A$.

b. Use that formula to determine when only $\frac{1}{3}$ of the initial amount will remain. (Leave your answer in terms of exponents and logs.)
Some useful formulas

Trig identities

\[ \cos^2 \theta = \frac{1 + \cos 2\theta}{2} \]
\[ \sin^2 \theta = \frac{1 - \cos 2\theta}{2} \]

Trig subs

\[
\begin{array}{|c|c|c|}
\hline
x &=& a \sin \theta \\
\frac{dx}{d\theta} &=& a \cos \theta \\
\sqrt{a^2 - x^2} &=& a |\cos \theta| \\
\hline
x &=& a \tan \theta \\
\frac{dx}{d\theta} &=& a^2 \sec^2 \theta \\
\sqrt{a^2 + x^2} &=& a |\sec \theta| \\
\hline
x &=& a \sec \theta \\
\frac{dx}{d\theta} &=& a \sec \theta \tan \theta \\
\sqrt{x^2 - a^2} &=& a |\tan \theta| \\
\hline
\end{array}
\]

Some useful integrals  Most of those on page 431 of the text you should know (especially 1–7). Here are some others

\[ \int \sec^2 x \, dx = \tan x + C \]
\[ \int \csc^2 x \, dx = - \cot x + C \]
\[ \int \sec x \tan x \, dx = \sec x + C \]
\[ \int \csc x \cot x \, dx = - \csc x + C \]
\[ \int \tan x \, dx = \ln |\sec x| + C \]
\[ \int \cot x \, dx = \ln |\sin x| + C \]
\[ \int \sec x \, dx = \ln |\sec x + \tan x| + C \]
\[ \int \csc x \, dx = - \ln |\csc x + \cot x| + C \]